RODOLFO GUIDOBONO | CURSO TEORÍA CUÁNTICA DE CAMPOS | JAVIER GARCÍA

EJERCICIO CAPÍTULO 64

**EJERCICIO 1**

**Considerando que**

$$\vec{\boldsymbol{E}} = \boldsymbol{E_0}\,\mathbf{cos}(\boldsymbol{kx})\,\hat{\boldsymbol{n}}$$

**Donde:**

$$\boldsymbol{kx} = \boldsymbol{\omega t} - \vec{\boldsymbol{k}}\cdot\vec{\boldsymbol{r}}$$

$$\vec{\boldsymbol{r}} = \boldsymbol{x}\,\hat{\boldsymbol{x}} + \boldsymbol{y}\,\hat{\boldsymbol{y}} + \boldsymbol{z}\,\hat{\boldsymbol{z}}$$

$$\vec{\boldsymbol{k}} = \boldsymbol{k_x}\hat{\boldsymbol{x}} + \boldsymbol{k_y}\hat{\boldsymbol{y}} + \boldsymbol{k_z}\hat{\boldsymbol{z}}$$

$$\hat{\boldsymbol{n}} = \boldsymbol{n_x}\hat{\boldsymbol{x}} + \boldsymbol{n_y}\hat{\boldsymbol{y}} + \boldsymbol{n_z}\hat{\boldsymbol{z}}$$

**Encontrar** $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}$

$$\vec{E} = \begin{pmatrix} E_0\cos(kx)\,n_x \\ E_0\cos(kx)\,n_y \\ E_0\cos(kx)\,n_z \end{pmatrix}$$

$$\vec{\nabla}\times\vec{\mathrm{E}} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_0\cos(kx)\,n_x & E_0\cos(kx)\,n_y & E_0\cos(kx)\,n_z \end{pmatrix}$$

$$\partial_x(E_0\cos kx) = E_0\,\partial_x\cos(\omega t - \vec{k}\cdot\vec{r}) = E_0\,\partial_x\cos(\omega t - (k_x x + k_y y + k_z z)) = E_0\,k_x\sin kx$$

$$\partial_y(E_0\cos kx) = E_0\,k_y\sin kx$$

$$\partial_z(E_0\cos kx) = E_0\,k_z\sin kx$$

$$\vec{\nabla}\times\vec{\mathrm{E}} = \begin{pmatrix} \partial_y(E_0\cos(kx)\,n_z) - \partial_z(E_0\cos(kx)\,n_y) \\ -\partial_x(E_0\cos(kx)\,n_z) + \partial_z(E_0\cos(kx)\,n_x) \\ \partial_x(E_0\cos(kx)\,n_y) - \partial_y(E_0\cos(kx)\,n_x) \end{pmatrix} = \begin{pmatrix} n_zE_0\,k_y\sin kx - n_yE_0\,k_z\sin kx \\ -n_zE_0\,k_x\sin kx + n_xE_0\,k_z\sin kx \\ n_yE_0\,k_x\sin kx - n_xE_0\,k_y\sin kx \end{pmatrix}$$

$$\vec{\nabla}\times\vec{\mathrm{E}} = E_0\sin kx\begin{pmatrix} n_z\,k_y - n_y\,k_z \\ -n_z\,k_x + n_x\,k_z \\ n_y\,k_x - n_x\,k_y \end{pmatrix}$$

Como

$$\vec{k}\times\hat{n} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ n_x & n_y & n_z \end{pmatrix} = \begin{pmatrix} k_yn_z - k_zn_y \\ -k_xn_z + k_zn_x \\ k_xn_y - k_yn_x \end{pmatrix}$$

$$\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}} = \boldsymbol{E_0}\,\mathbf{sin}(\boldsymbol{kx})(\vec{\boldsymbol{k}}\times\hat{\boldsymbol{n}})$$

**EJERCICIO 2**

**Dado**

$$\boldsymbol{A^\mu = \begin{pmatrix} 0 \\ a\sin kx \\ b\sin kx \\ c\sin kx \end{pmatrix}}$$

**Verificar si $\vec{\nabla} \times \vec{\mathrm{A}}$ reproduce el campo magnético**

$$\boldsymbol{\vec{A} = \begin{pmatrix} a\sin kx \\ b\sin kx \\ c\sin kx \end{pmatrix}}$$

$$\boldsymbol{\vec{\mathrm{B}} = E_0 \cos(kx)\left(\vec{k} \times \hat{n}\right)}$$

$$\vec{\nabla} \times \vec{\mathrm{A}} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ a\sin kx & b\sin kx & c\sin kx \end{pmatrix} = \begin{pmatrix} \partial_y(c\sin kx) - \partial_z(b\sin kx) \\ -\partial_x(c\sin kx) + \partial_z(a\sin kx) \\ \partial_x(b\sin kx) - \partial_y(a\sin kx) \end{pmatrix}$$

$$\partial_x(D\sin kx) = -Dk_x\cos kx$$

$$\partial_y(D\sin kx) = -Dk_y\cos kx$$

$$\partial_z(D\sin kx) = -Dk_z\cos kx$$

D es una constante igual a: a; b; c

$$\vec{\nabla} \times \vec{\mathrm{A}} = \begin{pmatrix} -c\,k_y\cos kx + b\,k_z\cos kx \\ +c\,k_x\cos kx - a\,k_z\cos kx \\ -b\,k_x\cos kx + a\,k_y\cos kx \end{pmatrix} = \begin{pmatrix} -c\,k_y + b\,k_z \\ +c\,k_x - a\,k_z \\ -b\,k_x + a\,k_y \end{pmatrix}\cos kx$$

$$\vec{\nabla} \times \vec{\mathrm{A}} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ a & b & c \\ k_x & k_y & k_z \end{pmatrix}\cos kx = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ -a & -b & -c \end{pmatrix}\cos kx = \vec{k} \times \vec{V}\cos kx$$

Donde:

$\vec{V} = \begin{pmatrix} -a \\ -b \\ -c \end{pmatrix}$ podemos hacer: $\hat{n} = \frac{1}{\sqrt{a^2+b^2+c^2}}\begin{pmatrix} -a \\ -b \\ -c \end{pmatrix}$

$$\vec{\nabla} \times \vec{\mathrm{A}} = \vec{k} \times \vec{V}\cos kx = \frac{\sqrt{a^2+b^2+c^2}}{\sqrt{a^2+b^2+c^2}}\vec{k} \times \vec{V}\cos kx = \sqrt{a^2+b^2+c^2}\left(\vec{k} \times\ \hat{n}\right)\cos kx$$

Como estamos considerando c = 1 entonces $\left|\vec{k}\right| = \omega$ resultando $\vec{k} = \left|\vec{k}\right| \cdot \hat{k} = \omega \cdot \hat{k}$

$$\vec{\nabla} \times \vec{\mathrm{A}} = \omega\sqrt{a^2+b^2+c^2}\left(\hat{k} \times\ \hat{n}\right)\cos kx$$

Javier había calculado: $E_0 = \omega\sqrt{a^2+b^2+c^2}$

$$\boldsymbol{\vec{\nabla} \times \vec{\mathrm{A}} = E_0\left(\hat{k} \times\ \hat{n}\right)\cos kx}$$

**EJERCICIO 3**

**Dado**

$$\boldsymbol{f = \alpha \sin kx}$$

**Demostrar que cumple**

$$\boldsymbol{\partial_\mu \, \partial^\mu f = 0}$$

**Considerar**

$$\boldsymbol{\partial^0 = \partial_0 \; ; \; \partial^a = -\partial_a \; para \; a = 1\,(x), 2(y), 3(z)}$$

$$f = \alpha \sin\left(\omega t - \vec{k} \cdot \vec{r}\right)$$

$$\partial_0 f = \omega \alpha \cos kx$$

$$\partial_0^2 f = -\omega^2 \alpha \sin kx$$

$$\partial_1 f = -k_x \, \alpha \cos kx$$

$$\partial_1^2 f = -{k_x}^2 \, \alpha \sin kx$$

$$\partial_2^2 f = -{k_y}^2 \, \alpha \sin kx$$

$$\partial_3^2 f = -{k_z}^2 \, \alpha \sin kx$$

$$\partial_\mu \; \partial^\mu \, f = \partial_0 \; \partial^0 \, f + \partial_1 \; \partial^1 \, f + \partial_2 \; \partial^2 \, f + \partial_3 \; \partial^3 \, f$$

$$\partial_\mu \; \partial^\mu \, f = \partial_0 \; \partial_0 \, f - \partial_1 \; \partial_1 \, f - \partial_2 \; \partial_2 \, f - \partial_3 \; \partial_3 \, f$$

$$\partial_\mu \; \partial^\mu \, f = -\omega^2 \alpha \sin kx + {k_x}^2 \, \alpha \sin kx + {k_y}^2 \, \alpha \sin kx + {k_z}^2 \, \alpha \sin kx$$

$$\partial_\mu \; \partial^\mu \, f = \left(-\omega^2 + {k_x}^2 + {k_y}^2 + {k_z}^2\right) \alpha \sin kx$$

Pero como c = 1 entonces $\left|\vec{k}\right| = \sqrt{{k_x}^2 + {k_y}^2 + {k_z}^2} = \omega$

Resultando

$$\boldsymbol{\partial_\mu \, \partial^\mu f = 0}$$